EXPERIMENT 4

THE RATIO OF HEAT CAPACITIES

Air contained in a large jar is subjected to an adiabatic expansion then an isochoric process. If the initial, final, and atmospheric pressures are known, then the ratio of heat capacities, γ, for air can be found.

THEORY

The heat capacity of a material is defined as the amount of heat that is absorbed or liberated by a material for a given change In temperature. Mathematically, this can be expressed as

$$\text{Heat capacity} = \frac{\Delta Q}{\Delta T}. \tag{1}$$

This expression is dependent upon the amount of material present. However, it is possible to eliminate any dependence upon the amount of material by dividing (1) by the mass m. or the number of moles m of the material. When divided by the mass, (1) becomes the specific heat of the material. or

$$\text{Specific heat} = c = \frac{1}{m}\frac{\Delta Q}{\Delta T}, \tag{2}$$

and when divided by the number of moles. (1) becomes the molar heat capacity:

$$\text{molar heat capacity} = C = Mc = \frac{1}{m}\frac{\Delta Q}{\Delta T}, \tag{3}$$

where M is the molecular mass of the material.

For gases, the heat that is exchanged between the system and the environment is greatly dependent upon whether the exchange of heat takes place isobarically or isochorically. Because of this a different value of the molar heat capacity must be specified for each process. In the former case, the molar heata capacity is written as C_p. and the latter as C_V. The ratio of the two values is known as the ratio of heat capacities. y. That is,

$$\gamma = \frac{C_p}{C_v}. \tag{4}$$

In this experiment, γ for air is determined from an observation of the pressure changes that take place when air undergoes an adiabatic expansion and an isochoric pressure increase.

Consider a gas enclosed in container at an initial pressure p_1, which is slightly greater than atmospheric pressure p_0. The initial temperature of the gas is room temperature T_R. Suppose that by momentarily operung a valve, the gas is allowed to attain atmospheric pressure. The change In pressure takes place so rapidly that no heat is considered to be transferred to or from the environment, and the expansion is assumed to be adiabatic. Because compressed gas In the container must perform'work forcing some of the gas out of the container, the temperature of the gas that remains in the container drops below room temperature. If the gas is now allowed to warm up to room temperature, the pressure increases to a final value of p_2.

In order to analyze this quantitatively. consider a container of volume V_0, divided into two parts by an imaginary membrane. (Refer to Figure 1.) Volume V_1, represents the volume that contains the N molecules that will remain in the container after the valve is opened. The Pressure in the container iS P2, and the temperature is room temperature T_R.
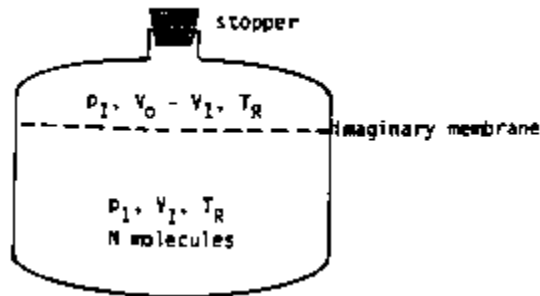

Figure 1. The container with the two volumes separated by an imaginary membrane.

Suppose that the valve is momentarily opened allowing the release of the gas above the membrane, as shown in Figure 2. The volume occupied by the N molecules is now V_0. The pressure in the container drops to atmospheric pressure p_0, and the temperature drops to T. below room temperature.
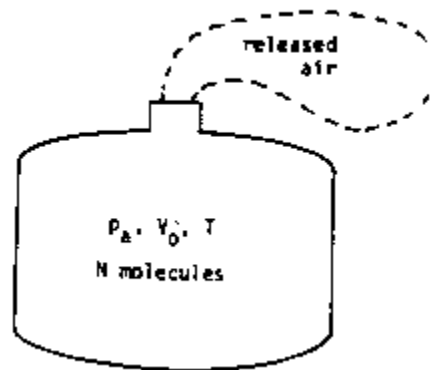

Figure 2. The container immediately after the adiabatic expansion.

Since the process is adiabatic,

$$p_1 V_1^v = p_0 V_0^v. \tag{5}$$

The pressure In the container gradually increases to p. as the gas in the container warms up to room temperature. (Refer to Figure 3.) When equilibrium is attained, the initial and final states of the gas containing the N molecules are related to each other as
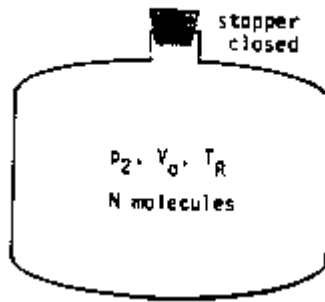
$$\frac{p_1 V_1}{T_R} = \frac{p_2 V_0}{T_R}. \tag{6}$$

Figure 3.   The final state of the gas in the container.

When the volumes are eliminated between (5) and (6), and the resulting equation solved for y. the result is

$$v = \frac{\ln(p_1 / p_0)}{\ln(p_1 / p_2)}. \tag{7}$$

Note that only pressures are needed in order to determine y.

The pressures, pi and P 2 , are determined using a manometer. (Refer to Figure 4.) It can be seen that the pressure inside the container is related to the heights of the mercury columns by

$$p = p_a + \rho g (h - h_o), \tag{8}$$

where h. is the height of the mercury column on the side of the manometer connected to the jar and h is the height on the opposite side. If both the atmospheric pressure and the height of the mercury columns are expressed in centimeters of mercury, then
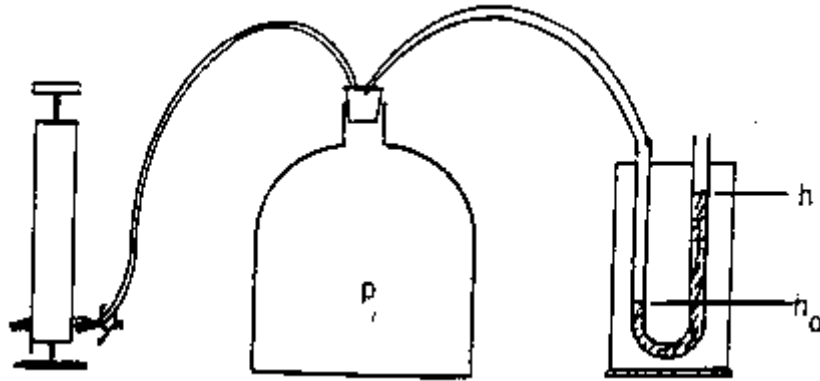
$$p = p_a + (h - h_o), \tag{9}$$

Figure 4. The experimental arrangement showing the pump, jar, and manometer.
The clamp is placed on the tubing between the pump and the jar.

APPARATUS

o	five gallon jar with stopper	o	pump
o	U-tube manometer	o	1 hose clamp

PROCEDURE

a) Connect the pump and the manometer to the stopper on the flve gallon jar. Slip the clamp on the tubing between the pump and the Jar.

b) Record the atmospheric pressure P,, and Its uncertainty.

c) Pump air into the Jar until the manometer shows a difference in heights of approximately 8 to 9 centimeters. Clamp the tube between the pump and the jar.

> CAUTION: Hold the rubber stopper so that it will not POP Off the jar while the ressure inside the Jar is being increased. Also be very careful not to tip the manometer over. Mercury spills

Wait until the mercury columns stop moving and record the height h, and h of each column and the uncertainty in the readings Ah.

d) Remove and replace the rubber stopper quickly. This is the adiabatic process. This is also the most difficult step in the procedure. If done too slowly, then heat transfer occurs; and if done to rapidly, then the gas in the container does not drop to atmospheric pressure.

e)   Wait for a couple Of minutes to a.Ilow the temperature of the gas to reach room temperature. then record the height of each mercury column.

f)   Loosen the clamp between the PUMP and the Jar.

g)   Repeat the above procedures four more times for a total of five trials.

h)   Repeat the experiment five more times, but pump air out of the container.

ANALYSIS

Instead of finding y and its uncertainty directly, the maximum value, y, and the minimum value, γ_{min}, are found. When air is pumped <u>into</u> the jar, the expressions are

$$\gamma_{max} = \frac{\ln[(p_1 + \delta p_1)/(p_0 - \delta p_0)]}{\ln[(p_1 - \delta p_1)/(p_2 + \delta p_2)]}, \tag{10}$$

and

$$\gamma_{min} = \frac{\ln[(p_1 - \delta p_1)/(p_0 + \delta p_0)]}{\ln[(p_1 + \delta p_1)/(p_2 - \delta p_2)]}, \tag{11}$$

However, when air is pumped out of the jar, (10) represents Y,,, and (11) represents y. Notice that the signs associated with the pressure uncertainties are chosen so as to maximize or minimize the quotients. The uncertainties in pressure Zip, and ap, are related as

$$\delta p_1 = \delta p_2 + \delta(h - h_0) = \delta p_0 + 2\delta p. \tag{12}$$

The best value for the ratio of heat capacities is

$$\gamma = \frac{\gamma_{max} + \gamma_{min}}{2}, \tag{13}$$

and the uncertainty is

$$\delta\gamma = \frac{\gamma_{max} - \gamma_{min}}{2}, \tag{14}$$

For each trial calculate y from (13) and the corresponding uncertainty Ay from (14). Report these values together with the best known value in a table. Also plot the values for each trial and the book value on a single one-dimensional graph with the values displaced vertically from each other.

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QUESTIONS

1. Draw the adiabatic and isochoric processes that are described above on a P-V diagram. Label the beginning and ending pc)ints of each of the processes with the pressure, volumes and temperature values from the discussion above.

2. Start with (5) and (6). and derive (7).

3. The handbook value of y for dr'y air Is 1.40. However, in Santa Monica moisture permeates everything. How does the moisture affect the value of γ? Explain.

4. On a pressure versus volume graph, draw the thermodynamic processes the air in this experiment went through. Label the graph with the numerical values of pressure, volume, and temperature at the three states. Assume that atmospheric pressure was 76 cm of Hg and room temperature was 220C. The volume of the rontainer V_0 is five gallons or 0.019 M3. Let $h - h_0 = 10$ cm for state 1.

5. Suppose the starting pressure in the container p, is below atmospheric pressure when the stopper is popped. Draw the adiabatic and isochoric processes on a p-V diagram.

6. How would y be affected if: (a) the adiabatic expansion was too rapid and the pressure in the container did not drop to atmospheric pressure; and (b) the adiabatic expansion was too slow so that a substantial amount of heat was transferred? Explain.